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MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Electric Currents in the Ionosphere. I. The Conductivity

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ELECTRIC CURRENTS IN THE IONOSPHERE

I. THE CONDUCTIVITY

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CONTENTS

An earlier suggestion by Martyn that the effective conductivity of the ionosphere in the dynamo theory is enhanced by polarization of the Hall current is examined in quantitative detail. General expressions are given for the conductivities of a thin ionized sheet oriented at an angle to a uniform magnetic field. The effective conductivity of such a (spherical) sheet surrounding the earth is shown to be greater than either the Pedersen or the Hall conductivities. The variation of conductivity with latitude is calculated for the ionospheric level of maximum effective conductivity. Consideration is given to the height-integrated conductivity of the actual ionosphere, and effective values deduced. It is shown that the F_2 region will move bodily under the influence of the electric field from lower regions, thereby reducing its ability to shunt the Hall polarization field.

The effective conductivity over most of the earth is found to be sufficient to satisfy Stewart's dynamo theory. In a narrow strip at the equator the conductivity is enhanced, thereby accounting for the anomalously large magnetic variations found to occur in these regions.

1. Introduction

It has long been known that the compass needle daily executes small regular oscillations. In a notable article Stewart (1882) reviewed critically certain explanations of the phenomenon suggested by Faraday and others, and concluded that the daily magnetic variations were due to electric currents in the *upper* atmosphere. Stewart also suggested that convective currents (of air) established by the sun's heating influence are to be regarded as 'conductors moving across lines of magnetic force, and are thus the vehicle of electric currents which act upon the magnet'. It is convenient to call this hypothesis of Stewart's the (atmospheric) 'dynamo theory'.

The theory received early support from Schuster (1889), who proved that the greater part of the field of the magnetic variations has its origin outside the earth, and that the remainder may reasonably be attributed to earth currents induced by the varying external field. Schuster (1908) later developed the dynamo theory in quantitative form. Chapman (1919) improved the theory further, giving special attention to the lunar magnetic variations, which are more amenable to theoretical treatment; he concluded that, if the lunar tidal air velocities are the same in the upper atmosphere as at the ground, then the (height-integrated) conductivity of the upper atmosphere, at sunspot maximum, in the region of the earth which has the sun at its zenith, must be about $2{\cdot}5 \times 10^{-5}$ e.m.u. cm.

The experimental discovery of the ionosphere, about 1925, by Appleton, and by Breit & Tuve, made it likely that the dynamo theory would soon be confirmed quantitatively. However, Pedersen (1927) drew attention to a serious reduction of conductivity which might be expected to occur in the ionosphere owing to the influence of the earth's magnetic field. At the low pressures there existing, ions and electrons execute spiral motions; their motion parallel to an electric field is seriously impeded, as was earlier demonstrated in the laboratory by Townsend; this might well reduce the calculated conductivity of the ionosphere at certain levels by a factor of 50 or more. Early calculations on this basis showed that the ionosphere was not likely to have a conductivity greater than about 5×10^{-9} e.m.u. cm, which is deficient by a factor of 5000 for the purposes of the dynamo theory.

For this reason various alternative theories, especially of the solar magnetic variations, were developed in the subsequent decade. These were shown by Cowling to be fallacious.

In 1936 the dynamo theory received a new impetus from the work of Taylor (1936) and Pekeris (1937) on atmospheric oscillations. Previous investigators had assumed two special temperature gradients: (*a*) isothermal, with isothermal expansions (Laplace); and (*b*) fully adiabatic, with adiabatic expansions (Lamb). In these two special cases, chosen by Laplace and Lamb for mathematical convenience, the amplitude of the oscillations is independent of height. It became clear from Taylor's work that these two cases were unique, while Pekeris showed that the amplitude tended to increase markedly with height. The physical principle involved in the general case appears to be that the horizontal *energy* flow tends to be uniform with height, save in the immediate vicinity of certain possible nodal regions. The energy flow is proportional to ρu^2, where ρ is the air density and u the speed of tidal motion. Thus u tends to vary as $\rho^{-\frac{1}{2}}$ and might be several thousand times greater in the ionosphere than at the ground, if damping be negligible.

This theoretical development appeared at first to receive experimental support from the discovery by Appleton & Weekes (1939) that the equivalent height of the E region in the ionosphere had a semi-diurnal lunar oscillation of half-range 1 km; this could be interpreted as showing that the lunar atmospheric oscillation at a height of 120 km was some 6000 times greater than at the ground. However, Martyn (1947) showed that this oscillation of the E region was likely to be due to the electrodynamic forces associated with the currents responsible for the lunar magnetic variations, and not to a simple tidal rise and fall of isobaric surfaces; this demanded much less amplification of tidal velocities, perhaps only 200. (It should be remarked also, for the case of the solar variations, that amplifications of 6000 would give tidal velocities greater than that of sound.)

It now appears, therefore, that in spite of the increase in tidal amplitude with height, the conductivity of the ionosphere is still deficient (for the purposes of the dynamo theory) by a factor of 5 to 10. This view is confirmed by the work of Ashour & Price (1948), and of Sugiura & Nagata (1949), who, from arguments based on current induction, conclude that the integrated conductivity of the ionosphere in the sunlit hemisphere is approximately 5×10^{-8} e.m.u. cm.

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In a theoretical discussion of the solar atmosphere Cowling (1933) considered the consequences, for the electron conductivity, of the inhibition of the transverse (Hall) current by polarization of the medium; he found this conductivity thereby increased from the Pedersen value to that which obtained in the absence of a magnetic field. Martyn (1948) suggested that this effect might be responsible for the high conductivity necessary in the ionosphere to meet the requirements of the dynamo theory. However, Cowling & Borger (1948) considered that Martyn's suggestion would be invalid save near the magnetic equator; at other latitudes the polarization necessary to prevent the flow of Hall current might be expected to leak away horizontally.

About this time (1948) it became known (Egedal 1947) that the daily magnetic variations are considerably enhanced in a narrow zone near the magnetic equators. Martyn (1949) examined the data accumulating regarding this equatorial enhancement, with a view to testing the applicability of the above ideas on enhanced conductivity to this region. He found evidence consistent with the existence of a narrow belt of high *conductivity* encircling the earth in a region lying near the equators (magnetic, geomagnetic and geographic). Martyn also showed that McNish's (1936) theory of these enhanced variations, which (effectively) attributed them to the non-coincidence of the earth's polar and (magnetic) dipole axes, could not be valid quantitatively.

At this stage it seemed clear that Martyn's suggestion regarding the Hall polarization was likely to account for the equatorial anomaly, but detailed quantitative investigation was necessary to establish its application to other parts of the earth. The present investigation was then started, with the object of examining in quantitative detail the effect of Hall polarization on the conductivity of the ionosphere in all latitudes. During the course of the investigation Hirono (1950) and Maeda (1951) have attacked the problem from the same viewpoint, in rather less quantitative detail.

In this paper, and in part II which follows, we have attempted a fairly complete solution of the dynamo problem, for a spherical sheet ionosphere of finite thickness. It is found that over the whole earth the effective conductivity appears to be greater than the Pedersen conductivity by a factor of at least 6 and, near the magnetic equator, is further increased by a factor of 2 to 5. It seems safe at last to assert that the dynamo theory, put forward by Stewart some seventy years ago, is valid, and that it accounts also for the equatorial anomaly, whose existence was unknown to Stewart and Schuster.

2. Conductivity of a lightly ionized gas

The most accurate way of calculating the conductivity of a gas is the 'velocity-distribution' method, originally developed by Chapman and Enskog, and applied to the ionosphere by Cowling (1945). It is clear from the work of the latter that for present purposes the simpler 'free-path' method of treatment will be adequate.

Consider an infinite volume of air in which there are N electrons and positive ions per cubic centimetre, N being much less than the volume density of neutral particles.

Let $m_{e,i}$ be the mass of an electron or ion,

e the electronic charge (considered as a positive number),

$\nu_{e,i}$ the electron or ion collisional frequency with neutral molecules,

H the intensity of a uniform magnetic field pervading the medium,

and put
$$\omega_{e,i} = He/m_{e,i},$$
$$\tan\alpha_{e,i} = \omega_{e,i}/\nu_{e,i},$$
$$A = Ne/H.$$

For an electric field applied parallel to the magnetic field the conductivity, which may be termed the 'longitudinal conductivity', is independent of H, and is

$$\sigma_0 = Ne^2\left(\frac{1}{m_e\nu_e}+\frac{1}{m_i\nu_i}\right). \tag{1}$$

This may be rewritten as

$$\sigma_0 = A(\tan\alpha_e+\tan\alpha_i). \tag{2}$$

For an electric field E applied perpendicularly to H the conductivity is

$$\sigma_1 = Ne^2\left\{\frac{\nu_e}{m_e(\omega_e^2+\nu_e^2)}+\frac{\nu_i}{m_i(\omega_i^2+\nu_i^2)}\right\}, \tag{3}$$

or
$$\sigma_1 = \tfrac{1}{2}A(\sin 2\alpha_e+\sin 2\alpha_i) = A\sin(\alpha_e+\alpha_i)\cos(\alpha_e-\alpha_i). \tag{4}$$

Equation (3) was deduced by Pedersen (1927), who pointed out its probable importance in the ionosphere; for clarity σ_1 is sometimes referred to below as the 'Pedersen' conductivity.

In the latter case there may also be a flow of electric current perpendicular to both E and H. This is the Hall current; because of the relative mobilities of electrons and ions it has the direction of the vector product $\mathbf{H}\times\mathbf{E}$.

The current in this direction may be specified quantitatively by the 'Hall' conductivity,

where
$$\sigma_2 = Ne^2\left\{\frac{\omega_e}{m_e(\omega_e^2+\nu_e^2)}-\frac{\omega_i}{m_i(\omega_i^2+\nu_i^2)}\right\}, \tag{5}$$

or
$$\sigma_2 = A(\sin^2\alpha_e-\sin^2\alpha_i) = A\sin(\alpha_e+\alpha_i)\sin(\alpha_e-\alpha_i). \tag{6}$$

In the general case $\mathbf{E}$ may be resolved into components $\mathbf{E}_0$ and $\mathbf{E}_1$ respectively parallel and perpendicular to H. If $\mathbf{h}$ is a unit vector parallel to $\mathbf{H}$ the current density $\mathbf{J}$ is given by

$$\mathbf{J} = \sigma_0\mathbf{E}_0+\sigma_1\mathbf{E}_1+\sigma_2(\mathbf{h}\times\mathbf{E}). \tag{7}$$

3. Conductivity of a thin ionized sheet

The ionosphere may be regarded as a thin spherical conducting sheet enclosing the earth. The conductivity of an infinite medium was considered in § 2; from now on the discussion refers mainly to ionized sheets, in which the flow of current is restricted to the plane of the sheet, although there will usually be a component of E perpendicular to the sheet.

Consider a plane sheet lying in the x, y-plane. The magnetic field dips downward at an angle χ to the plane, and the x- and y-axes coincide with magnetic south and east respectively. Then if the z-direction be considered as pointing upwards, the sheet can be considered as a portion of the ionosphere in the northern hemisphere.

Let i, j, k be unit vectors in the x-, y-, z-directions respectively, then the magnetic field is represented by

$$\mathbf{H} = H\mathbf{h} = -H\cos\chi\mathbf{i}-H\sin\chi\mathbf{k}, \tag{8}$$

while the electric field is

$$\mathbf{E} = E_x\mathbf{i}+E_y\mathbf{j}+E_z\mathbf{k}. \tag{9}$$

The components of **E** parallel and perpendicular to **H** are, respectively,

$$\mathbf{E}_0 = \mathbf{Ehh} = (E_x \cos^2\chi + E_z \cos\chi \sin\chi)\,\mathbf{i} + (E_x \cos\chi \sin\chi + E_z \sin^2\chi)\,\mathbf{k}, \tag{10}$$

$$\mathbf{E}_1 = (E_x \sin^2\chi - E_z \cos\chi \sin\chi)\,\mathbf{i} + E_y\,\mathbf{j} + (-E_x \cos\chi \sin\chi + E_z \cos^2\chi)\,\mathbf{k}. \tag{11}$$

The vector product necessary for calculating the Hall current is

$$\mathbf{h} \times \mathbf{E} = E_y \sin\chi\,\mathbf{i} + (-E_x \sin\chi + E_z \cos\chi)\,\mathbf{j} - E_y \cos\chi\,\mathbf{k}. \tag{12}$$

Thus the current density is

$$\begin{aligned}\mathbf{J} = \{&(\sigma_0 \cos^2\chi + \sigma_1 \sin^2\chi)\,E_x + \sigma_2 E_y \sin\chi + (\sigma_0 - \sigma_1)\,E_z \cos\chi \sin\chi\}\,\mathbf{i} \\ &+ \{\sigma_1 E_y - \sigma_2(E_x \sin\chi - E_z \cos\chi)\}\,\mathbf{j} \\ &+ \{(\sigma_0 - \sigma_1)\,E_x \cos\chi \sin\chi - \sigma_2 E_y \cos\chi + (\sigma_0 \sin^2\chi + \sigma_1 \cos^2\chi)\,E_z\}\,\mathbf{k}.\end{aligned} \tag{13}$$

Since the vertical component of current is normal to the sheet it must be zero, so

$$E_z = \{\sigma_2 E_y \cos\chi - (\sigma_0 - \sigma_1)\,E_x \cos\chi \sin\chi\}/(\sigma_0 \sin^2\chi + \sigma_1 \cos^2\chi). \tag{14}$$

Eliminating E_z from equation (13) gives

$$\left.\begin{aligned} J_x &= \sigma_{xx} E_x + \sigma_{xy} E_y, \\ J_y &= -\sigma_{xy} E_x + \sigma_{yy} E_y, \end{aligned}\right\} \tag{15}$$

where

$$\sigma_{xx} = \sigma_0 \sigma_1/(\sigma_0 \sin^2\chi + \sigma_1 \cos^2\chi), \tag{16}$$

$$\sigma_{xy} = \sigma_0 \sigma_2 \sin\chi/(\sigma_0 \sin^2\chi + \sigma_1 \cos^2\chi), \tag{17}$$

$$\sigma_{yy} = \{\sigma_1 \sigma_0 \sin^2\chi + (\sigma_1^2 + \sigma_2^2) \cos^2\chi\}/(\sigma_0 \sin^2\chi + \sigma_1 \cos^2\chi). \tag{18}$$

In the special case $\chi = 0$ (the magnetic equator)

$$\sigma_{xx} = \sigma_0, \quad \sigma_{xy} = 0, \quad \sigma_{yy} = (\sigma_1^2 + \sigma_2^2)/\sigma_1 \tag{19}$$

while at the pole ($\chi = 90°$) $\quad \sigma_{xx} = \sigma_{yy} = \sigma_1, \quad \sigma_{xy} = \sigma_2.$ (20)

When $\sigma_0 > 2\sigma_1$, which is the case in all regions of the ionosphere save the lower *D* region, σ_{xy} has a maximum, for a given height, at the latitude where the dip angle is given by

$$\sin^2\chi = \sigma_1/(\sigma_0 - \sigma_1). \tag{21}$$

When $\sigma_1 \cos^2\chi \ll \sigma_0 \sin^2\chi$, which is generally true save at the magnetic equator and/or in the lower *D* region,

$$\sigma_{xx} = \sigma_1 \operatorname{cosec}^2\chi, \quad \sigma_{xy} = \sigma_2 \operatorname{cosec}\chi, \quad \sigma_{yy} = \sigma_1 + \frac{\sigma_2^2}{\sigma_0} \cot^2\chi. \tag{22}$$

Equations (15) to (18) determine the current in an ionized sheet for given E_x and E_y. In any specific problem, however, it will usually be found that E_x and E_y are not independent. For example, a field E_x applied from electrodes or by induction may result in the appearance of a polarization field E_y, owing to the total or partial inhibition of the Hall current by the boundaries of the sheet; this field will necessarily react on the current flow in the *x*-direction. It is obviously important to know the 'effective' conductivity σ defining the flow of current in the direction of, and in terms of, the applied field. By equation (15) this is

$$\sigma = \frac{J_x}{E_x} = \sigma_{xx} + \frac{\sigma_{xy}^2}{\sigma_{yy}} + \frac{\sigma_{xy} J_y}{\sigma_{yy} E_x}. \tag{23}$$

If the circumstances of the problem are such that the Hall current builds up no polarization, then $E_y = 0$, $J_y = -\sigma_{xy} E_x$, so that $\sigma = \sigma_{xx}$. Hence when the Hall current is uninhibited the effective conductivity of the medium is the Pedersen conductivity. Such a case would occur, for example, if electrodes were applied at the north and south poles of the (spherical sheet) ionosphere. The applied field would then lie along the meridians, and Hall current would flow round the parallels of latitude; no electrostatic field could develop around the latter, as is evident on simple grounds of symmetry. The current flows in spirals from pole to pole.

If, on the other hand, the Hall current is entirely prevented from flowing by the boundaries, then $J_y = 0$, and $\sigma = \sigma_{xx} + \sigma_{xy}^2/\sigma_{yy}$. Since σ_{xy} can be considerably greater than either σ_{xx} or σ_{yy} it follows that σ may be much greater than any of the conductivities of equations (22). An example of such a case would be provided by the diverging flow of material from a point source in the sheet. By 'dynamo' action this would create a current around the source; the Hall current initially would flow parallel to the stream of material, thus building up electric charge of one sign at the source, and a distant diffused charge of opposite sign; the resultant field would eventually prevent the flow of Hall current, and could greatly increase the effective conductivity of the sheet for the circulatory current. Another example is provided by the 'dynamo theory' of the currents produced by tidal circulation of air in the ionosphere, as discussed more fully below.

4. The dynamo theory, taking account of Hall conductivity

An outline of the 'dynamo theory' of the magnetic variations has been given by Chapman & Bartels (1940). At latitudes above about 35° tidal winds in the ionosphere, blowing horizontally across the vertical component of the earth's magnetic field, generate currents which tend to flow at certain longitudes towards the equator, away from it along other meridians. These currents almost instantaneously build up a polarization field of potential S which, at low latitudes, permits closure of the current system by flow along the parallels of latitude. In the quasi-steady state the current flows without divergence, and a current function exists, strictly analogous to the stream function of hydrodynamics. Thus, let u, v be the southward and eastward components of the air velocity in the ionosphere, then the 'dynamo' fields are

$$E_x = vH_z, \quad E_y = -uH_z. \tag{24}$$

Taking the (electric) current function as R, then in polar co-ordinates

$$J_x = \frac{\partial R}{a \sin\theta\, \partial\phi}, \quad J_y = -\frac{\partial R}{a\, \partial\theta}, \tag{25}$$

a being the radius of the earth.

Let S be the electrostatic potential of the charge distribution set up over the earth; this gives rise to fields

$$E'_x = -\frac{\partial S}{a\, \partial\theta}, \quad E'_y = -\frac{\partial S}{a \sin\theta\, \partial\phi}. \tag{26}$$

In general σ_{xx} and σ_{yy} (equations (22)) differ from σ_1 by a factor of order unity; similarly, σ_{xy} is not very different from σ_2 save close to the magnetic equator; for simplicity here we adopt the value σ_1 for both the former, and σ_2 for the latter conductivities. (The complications arising close to the equator will be discussed in part II.) Then by Ohm's law

$$\frac{\partial R}{a\sin\theta\,\partial\phi} = \sigma_1\left(vH_z - \frac{\partial S}{a\,\partial\theta}\right) - \sigma_2\left(uH_z + \frac{\partial S}{a\sin\theta\,\partial\phi}\right), \tag{27}$$

$$\frac{\partial R}{a\,\partial\theta} = \sigma_1\left(uH_z + \frac{\partial S}{a\sin\theta\,\partial\phi}\right) + \sigma_2\left(vH_z - \frac{\partial S}{a\,\partial\theta}\right). \tag{28}$$

Eliminating S from these two equations gives

$$\frac{\partial^2 R}{\partial\theta^2} + \cot\theta\frac{\partial R}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2 R}{\partial\phi} = \frac{a(\sigma_1+\sigma_2^2/\sigma_1)}{\sin\theta}\left\{\frac{\partial(uH\sin\theta)}{\partial\theta} + \frac{\partial(vH_z)}{\partial\phi}\right\}. \tag{29}$$

This equation is identical with that of the dynamo theory (Chapman & Bartels 1940), save that the conductivity K of the latter is replaced by $(\sigma_1+\sigma_2^2/\sigma_1)$. It appears, therefore, in the light of the discussion in § 3, that the Hall current is inhibited, and that the effective conductivity is greater than the Pedersen conductivity; the extent of the latter enhancement is considered below.

5. Conductivity in the ionosphere

The formulae developed in §§ 2, 3 for the conductivity of an ionized gas involve the collisional frequencies of electrons and ions with neutral molecules; they therefore depend on the pressure of the gas. According to classical kinetic theory all particles would have the same average kinetic energy, and their r.m.s. velocities should be inversely as the square roots of their masses. With a Maxwellian distribution, the mean velocities would be in the same ratio. The free path for electrons should be longer than that for a heavy particle by a factor of $4\sqrt{2}$. The gyro-frequency ω is inversely proportional to the mass of the particle, thus $\tan\alpha_e/\tan\alpha_i$ should be independent of temperature, and, if the positive ions are mostly atomic oxygen, should be about 1000.

Modern theory shows that the electrons may have a higher effective temperature than the positive ions, and a non-Maxwellian distribution. The effective free path is determined by quantum considerations. In the light of present knowledge (Bates & Massey 1951) it appears reasonable to assume $\tan\alpha_e/\tan\alpha_i = k^2 = 1650$ throughout the important conducting regions of the ionosphere.

By equations (4) and (6) we have

$$\sigma_2/\sigma_1 = \tan(\alpha_e - \alpha_i) = \tan\alpha_i(k^2-1)/(1+k^2\tan^2\alpha_i). \tag{30}$$

This ratio has its maximum when $\tan\alpha_i = 1/k$, its value then being

$$(\sigma_2/\sigma_1)\max = (k^2-1)/2k \doteqdot 20. \tag{31}$$

At this pressure $(\sigma_1^2+\sigma_2^2)/\sigma_1$ is also a maximum, with a value of approximately 400, while σ_0/σ_1 is then 800.

At this level σ_{xx}, σ_{xy} and σ_{yy}, although reasonably constant at high latitudes, vary rapidly when the magnetic dip is small. For a dipole field, the relation between dip and (geomagnetic) latitude is

$$\tan\chi = 2\tan\Phi. \tag{32}$$

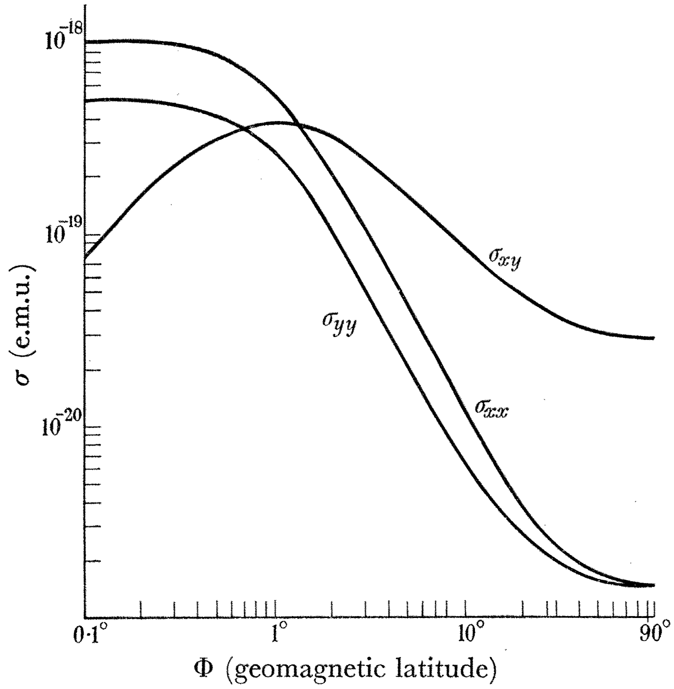

FIGURE 1. Variation with geomagnetic latitude (Φ), per electron-ion pair, of the horizontal ionospheric conductivities σ_{xx} (Pedersen, north-south), σ_{yy} (Pedersen, east-west) and σ_{xy} (Hall) at the height of maximum effective conductivity. The scale of Φ is logarithmic, to show the variations at low latitudes.

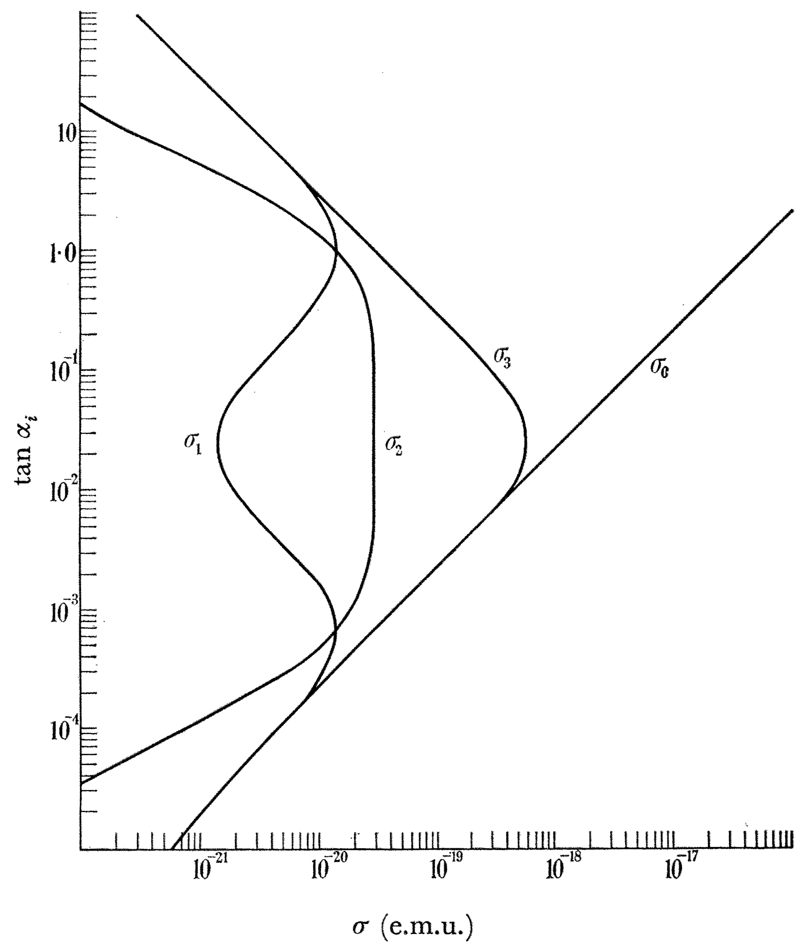

FIGURE 2. Variation of conductivities σ_0 (longitudinal), σ_1 (Pedersen), σ_2 (Hall), σ_3 (effective with Hall current inhibited), per electron-ion pair, plotted against tan α_i $(=\omega_i/\nu_i)$.

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In figure 1 is shown the variation of σ_{xx}, σ_{xy}, σ_{yy} for this level, Φ being on a logarithmic scale of latitude, so that the rapid changes near the magnetic equator may be more readily assessed. It will be seen that σ_{xy} is much the largest conductivity, except in the immediate vicinity of the equator.

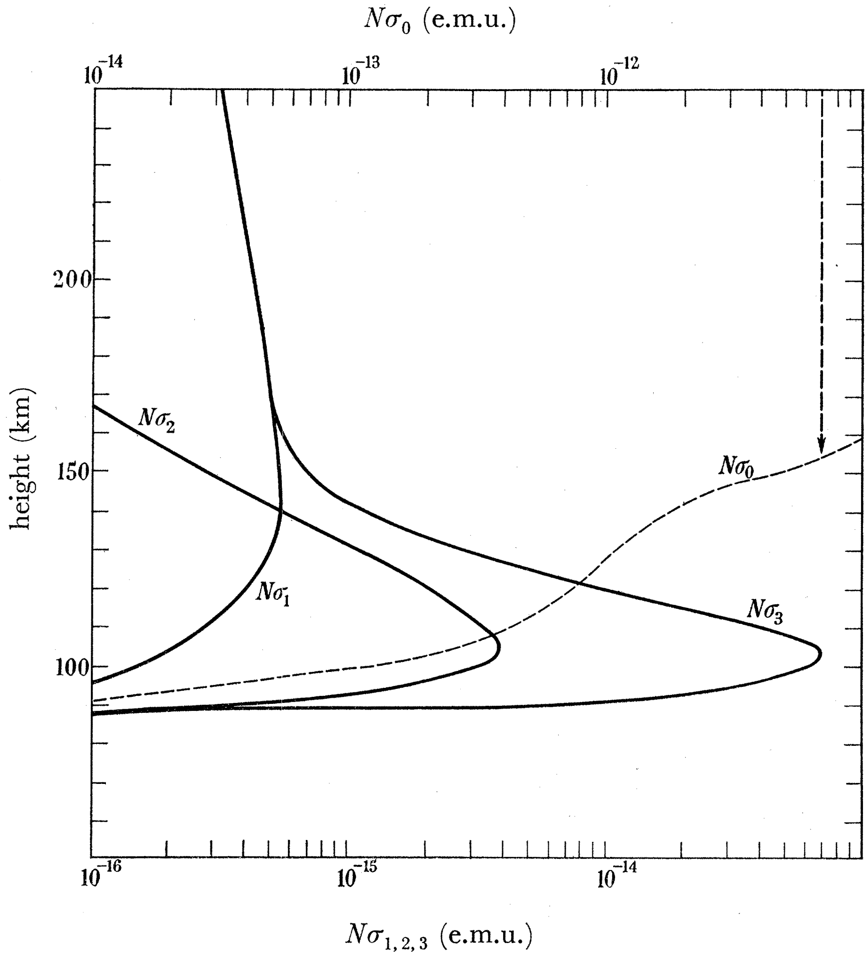

FIGURE 3. Variation with height of calculated conductivities for assumed electron density distribution.

It follows that at this height the current flow is substantially perpendicular to the electric field. Since equation (29) shows that the current system coincides in direction with that deduced from the classical 'dynamo theory' it follows, if this be the main conducting region, that the total electric field is substantially perpendicular to that deduced in former treatments of the dynamo theory (Schuster 1908; Chapman 1919). This result must have important consequences for the theory of tidal vertical drift of ionospheric regions under electrodynamic action.

At other levels in the ionosphere the ratios of σ_2/σ_1, etc., are not so large. There are two maxima for σ_1, near the levels where $\tan\alpha_e$ and $\tan\alpha_i$ respectively become unity. The variations of σ_0, σ_1, σ_2 and σ_3 (i.e. $\sigma_1+\sigma_2^2/\sigma_1$) against $\tan\alpha_i$, for one ion pair per unit volume, are shown in figure 2.

To proceed further, knowledge is necessary of the distribution of air pressure, and of electron (and ion) density with height. The best available value of ν_e at 100 km appears to derive from measurements on the interaction of radio waves (Shaw 1951), and is $2{\cdot}5\times10^5$ collisions per second. Bates & Massey (1951) summarize the best available estimates of air densities in the ionosphere. The variation with height of N, the electron (and ion) density is known reasonably well, though there is necessarily some uncertainty about the depth of the trough between the maxima in E and F_1. We have taken Mitra's (1952) estimates of N as adequately representative for these maxima, at sunspot maximum, in moderately high latitudes. The calculated height distributions of $N\sigma_{0,1,2,3}$ are shown in figure 3. (Here σ_3 is written for $\sigma_1+\sigma_2^2/\sigma_1$.) Since a sharp maximum of σ_1 occurs near 140 km. uncertainty regarding the height distribution of N between E and F_1 is more important for this conductivity than for the others. In computing σ_1 for figure 3 it has been assumed that a minimum of N occurs at 150 km, of magnitude 3×10^4 electrons/cm³. The consequences of assuming other minimum distributions is discussed below.

6. Height-integrated currents in a sheet of finite thickness

The ionosphere is thin with respect to the radius of the earth, but its thickness cannot be considered as infinitesimal. It has been found in § 5 that the various conductivities, and especially σ, the effective conductivity, vary markedly with height. It is necessary therefore to give careful attention to the effective *integrated* conductivity. The dominant conductivity is σ_0, so that the (usually nearly vertical) lines of magnetic force are nearly equipotentials. Thus all regions are linked together electrically, and a region of high σ (e.g. the E region) may be shunted by one (F) of appreciable σ_1; this shunting could *reduce* the effective value of σ over the whole thickness of the ionosphere. (The physical reason for this reduction is that the field necessary to prevent the flow of Hall current becomes less as σ_1 increases.)

The total field at any level is made up of two parts, the dynamo field E and the electrostatic field E' (§ 4). Since $E_x = H_z v$ and $E_z = -H_x v$ it follows that

$$E_z/E_x = -H_x/H_z = -\cot\chi.$$

Moreover, since the lines of magnetic force are nearly equipotentials, the gradient of S is perpendicular to them, hence

$$E'_z/E'_x = -\cot\chi$$

and

$$E^0_z/E^0_x = (E_z+E'_z)/(E_x+E'_x) = -\cot\chi.$$

The current will be horizontal in this part of the sheet if equation (14) be satisfied. Unless χ is very small all terms save those in σ_0 are negligible in this equation, which reduces to

$$E^0_z/E^0_x = -\cot\chi.$$

Thus, except in a narrow strip at the equator, the current in the ionosphere will always be horizontal, *whatever be the variation of tidal velocity with height.*

If we now confine our attention to tidal velocities independent of height throughout the important conducting region of the ionosphere, it is possible to derive the effective height-integrated conductivity, so permitting a two-dimensional treatment of current problems. If 'h' be height above the base of the ionosphere, then equations (15) give

$$\left.\begin{aligned} I_x &= \int J_x\,\mathrm{d}h = \int \sigma_{xx} E^0_x\,\mathrm{d}h + \int \sigma_{xy} E^0_y\,\mathrm{d}h,\\ I_y &= \int J_y\,\mathrm{d}h = -\int \sigma_{xy} E^0_x\,\mathrm{d}h + \int \sigma_{yy} E^0_y\,\mathrm{d}h.\end{aligned}\right\} \tag{33}$$

Since E'_x is derived from a potential, it follows that $\partial E'_x/\partial z = \partial E'_z/\partial x$. The latter derivative is necessarily small, since it expresses the variation of E'_z horizontally over the earth, so E'_x (and E'_y) will be almost constant throughout the ionosphere above a given point. It is postulated above that E_x and E_y also have negligible height gradient; hence E^0_x and E^0_y may be taken out of the integrals.

For meridional currents ($I_y = 0$),* we can eliminate E^0_y from equations (33), and the effective height-integrated conductivity is then

$$\Sigma_x = I_x/E^0_x = \int \sigma_{xx}\,\mathrm{d}h + \left(\int \sigma_{xy}\,\mathrm{d}h\right)^2 \Big/ \int \sigma_{yy}\,\mathrm{d}h. \tag{34}$$

Similarly the height-integrated east-west conductivity is

$$\Sigma_y = \int \sigma_{yy}\,\mathrm{d}h + (\sigma_{xy}\,\mathrm{d}h)^2/\sigma_{xx}\,\mathrm{d}h. \tag{35}$$

At the poles both these expressions simplify to

$$\Sigma_x = \Sigma_y = \int \sigma_1\,\mathrm{d}h + \left(\int \sigma_2\,\mathrm{d}h\right)^2 \Big/ \int \sigma_1\,\mathrm{d}h. \tag{36}$$

At the magnetic equator

$$\Sigma_x = \int \sigma_0\,\mathrm{d}h, \tag{37}$$

$$\Sigma_y = \int (\sigma_1 + \sigma_2^2/\sigma_1)\,\mathrm{d}h = \int \sigma_3\,\mathrm{d}h. \tag{38}$$

Equations (36) and (38) are superficially similar; in the latter, however, the mode of integration gives weight to the high values of σ_2^2/σ_1 near the 100 km level, which considerably enhance the integrated conductivity at the equator.

The integrated conductivities of the ionosphere, for the distributions given in figure 3, are (in e.m.u. cm)

$$\int \sigma_1\,\mathrm{d}h = 6{\cdot}4\times 10^{-9}, \quad \int \sigma_2\,\mathrm{d}h = 1{\cdot}36\times 10^{-8}, \quad \int \sigma_3\,\mathrm{d}h = 1{\cdot}64\times 10^{-7}.$$

Thus at the poles the integrated conductivity Σ assumed effective in the dynamo theory is $3{\cdot}5\times 10^{-8}$ e.m.u. cm; in a narrow belt at the equator it is $1{\cdot}64\times 10^{-7}$ e.m.u. cm. The conductivity over the rest of the earth, save in low latitudes, is approximately

$$\Sigma\,(\text{polar})\,\operatorname{cosec}^2\chi.$$

In computing the curves of figure 3 an ionization minimum was assumed at a height of 150 km. The position and depth of this trough have little effect on $\int \sigma_2\,\mathrm{d}h$, or even on $\int \sigma_3\,\mathrm{d}h$, but are important for $\int \sigma_1\,\mathrm{d}h$, since this conductivity has a maximum near 140 km. It is clear that N at these levels cannot be greater than its maximum value in the E region, and that the greatest value of $\int \sigma_1\,\mathrm{d}h$ will be obtained by assuming that N between E and F_1 is only slightly less than $1{\cdot}5\times 10^5$ electrons/cm³. The values of Σ computed for such a dis-

* It will be noted that even when the height-integrated current is zero, equation (33) (for I_y) shows that there will be in general a finite current density (J_y) at any selected height. Thus the current must flow in opposing directions over certain ranges of height.

tribution will be the lowest possible; they are found to be $2{\cdot}12\times10^{-8}$ e.m.u. cm at the poles and $4{\cdot}61\times10^{-8}$ e.m.u. cm at the equator.

In § 7 below it is shown that the effective value of σ_1 in the F_2 region is likely to be less than that calculated above, since the whole F_2 region is set in motion by the applied electric field. We can make a rough allowance for this effect by calculating Σ on the assumption that σ_1 is zero from a height of 200 km upwards. Assuming this, for the distribution of figure 3, gives Σ (polar) $= 4{\cdot}72\times10^{-8}$ e.m.u. cm, and Σ (equatorial strip) $= 1{\cdot}64\times10^{-7}$ e.m.u. cm.

It seems clear that for any likely distribution of N the effective integrated conductivity of the ionosphere over most of the earth is between 5 and 10 times greater than that calculated by Pedersen's formula, and that it is further enhanced in a narrow strip near the equator by a factor between 2 and 5. It appears that evidence is accumulating from rocket ascents* that the previously assumed air densities in the ionosphere may be over-estimated. If these preliminary conclusions are substantiated, σ_1 will be further reduced in the upper regions of the ionosphere, and Σ further increased.

Summing up, the evidence as a whole suggests that the conductivity of the ionosphere is adequate for the purposes of the dynamo theory, and that its additional enhancement at the equator provides a logical explanation of the anomalously large magnetic variations in these regions. A quantitative treatment of the theory of these variations will be given in part II.

Over most of the earth the currents producing the magnetic variations will flow transversely to the electric field in the E region; at higher levels there will be a component of current parallel to the field. At the magnetic equator the entire current system will be strongly concentrated near the 100 km level and will flow parallel to the electric field. In the latter connexion it is interesting to note that a rocket-borne magnetometer has recently been projected into the ionosphere near the magnetic equator (Singer, Maple & Bowen 1951). These authors report that the rocket passed through a region, between 93 and 105 km, in which the current system was sufficiently intense to produce the observed large magnetic variations at ground level.

7. The effective conductivity of the F region for electric fields of semi-diurnal periodicity

In the foregoing calculations of ionospheric conductivities it has been supposed that the milieu of neutral air particles is sufficiently dense to withstand reaction from the movements of the charged particles. It has been tacitly assumed, for example, that the polarization field communicated from lower regions would produce currents in the F_2 region whose magnitude would be given by equations (22). In this region the Hall conductivity is negligible, and the contributions made by σ_{yy} and σ_{xx} to the integrals in equations (34) and (35) tend to reduce the effective height-integrated conductivities of the whole ionosphere; in effect the predominantly 'Pedersen' conductivity of the F_2 region reduces the magnitude of the Hall polarization field built up in lower regions. The total current flow, which is mainly in the lower regions, is substantially proportional to the Hall field, and is correspondingly *reduced* by the 'shunting' effect of the F_2 region. The following analysis shows

* Private communication.

that this 'shunting effect' is less than might be expected; it appears that the whole F_2 region, both air and ionization, will be set in motion by the field from below, and will not draw off the expected current, just as an electric motor accepts, from a source of fixed voltage, less current when moving than when stationary.

It will suffice for present purposes to consider the meridional field E'_x, and to suppose it a simple harmonic function of time, say $E'_x = E_0 \cos \omega t$. The resulting motion of the F_2 region will be directed eastward (in the northern hemisphere) with velocity, say, $\dot{y}$. Owing to this motion of the medium there will be a local 'dynamo' field directed southwards. Thus the total effective field on the F_2 region will be $E_0 \cos \omega t + H_z \dot{y}$. The differential equation for the motion of the medium is then

$$\rho \ddot{y} = -j_x H_z, \tag{39}$$

where j_x is the south current density in the medium, whose density is ρ. This equation, which states that the horizontal force acting on a cubic centimetre of medium is the Ampere force due to interaction of current and magnetic field, may be rewritten

$$\ddot{y} + \frac{\sigma_{xx} H_z^2}{\rho} \dot{y} = -\frac{\sigma_{xx} H_z}{\rho} E_0 \cos \omega t. \tag{40}$$

The solution (for $\dot{y}$) is

$$\dot{y} = -\frac{\sigma_{xx} H_z}{(\rho^2\omega^2 + \sigma_{xx}^2 H_z^4)^{\frac{1}{2}}} E_0 \sin(\omega t - \alpha), \tag{41}$$

with $\tan \alpha = -\sigma_{xx} H_z^2/\rho\omega$. Thus the field in the F_2 region is

$$E_0 \cos \omega t + H_z \dot{y} = E_0 \left\{ \cos \omega t - \frac{\sigma_{xx} H_z^2}{(\rho^2\omega^2 + \sigma_{xx}^2 H_z^4)^{\frac{1}{2}}} \sin(\omega t - \alpha) \right\}. \tag{42}$$

In the F_2 region, for diurnal (or semi-diurnal) field variations $\sigma_{xx} H_z^2 > \rho\omega$. Thus α approaches $-\frac{1}{2}\pi$ and the two terms on the right side of (42) are comparable in magnitude and of opposite sign; the field effective in producing current in the F_2 region is considerably less than that applied from the lower region, and leakage of Hall polarization is correspondingly reduced.

Precise evaluation of this effect is difficult, since account should be taken of the free periods of oscillation of the atmosphere; this may introduce a resonance term in (40). However, it is clear that the effective height-integrated conductivity of the ionosphere will be greater than that calculated from (34) and (35), since the contribution of the F_2 region to $\int \sigma_{xx} \, dh$ and $\int \sigma_{yy} \, dh$ should be reduced.

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